Problem 1: Rotational symmetry (4 points):
A lattice has a n-fold rotation axis. The vector $\mathbf{a}$ denotes a primitive translation in the plane perpendicular to the n-fold rotation axis. Rotation of $\mathbf{a}$ by $\pm \frac{2\pi}{n}$ results in the vectors $\mathbf{a}^+$ and $\mathbf{a}^-$. Show that the sum $\mathbf{a}^+ + \mathbf{a}^-$ is an integral multiple ('ganzzahliges Vielfaches') of the vector $\mathbf{a}$. By using this result show that n can only posses the values 1, 2, 3, 4, 6.

Problem 2: Copper-oxygen planes (4 points):

a) All high-temperature superconductors possess copper-oxygen planes as key component of their crystal lattice. Black and white circles in the drawing denote copper and oxygen atoms, respectively. The Cu-Cu distance is $a$. For simplicity we consider the two-dimensional case. What is the rotational symmetry of the lattice? Sketch the Bravais lattice, give a pair of primitive lattice vectors and denote the unit cell and the basis.

b) In LaCuO$_4$ the copper-oxygen planes are not quite flat. The oxygen ions are displaced to positions slightly below (-) or above (+) the plane. Denote the rotation symmetry, the Bravais lattice and the primitive cell as in a). Is $a$ still the value of the lattice constant?

Problem 3: Lattice properties (4 points):
Calculate the following characteristic values for the cubic face and body centred the hexagonal closed-packed and diamond lattice:

a) Number K of nearest neighbours
b) Number Z of atoms per unit cell
c) Distance d to nearest-neighbour atoms in units of the lattice vector $\mathbf{a}$
d) The density $R = Z \cdot V / V_0$, with $V_0$ as the volume of the unit cell and $V_\lambda = (4/3) \pi (d/2)^3$ as the Volume of a single atom

Problem 4: Laue diffraction (4 points):
Analyze the Laue diffractogram below. What kind of symmetry operations do you find?