Problem 1: Excitons (4 points):

From the lectures you know that in a semiconductor a pair of an electron and a hole, a so called exciton, can be described like a hydrogen atom. The figure below shows the optical absorption spectrum of Cu2O (relative dielectric constant 7.5). Determine the gap energy and the reduced mass and Bohr radius of the exciton.

Hint: Use the peak positions in the spectrum!

Problem 2: Dielectric constant of an ionic crystal (6 points):

Show that the dielectric constant $\varepsilon$ of an ionic crystal as a function of the frequency $\omega$ is given by:

$$
\varepsilon = \frac{\omega_T^2 - \omega^2}{\omega_T^2 - \omega_L^2}
$$

$\omega_L$ and $\omega_T$ are the longitudinal and transversal, respectively, eigenfrequencies of the crystal lattice defined as:

$$
\omega_L = \omega_0 \sqrt{1 + \frac{4}{3} \alpha_{el}(0)}
$$

and

$$
\omega_T = \omega_0 \sqrt{1 - \frac{4}{3} \alpha_{ion}(0) / \alpha_{el}(0)}
$$

$\omega_0$ the frequency of the optical lattice oscillation for $k=0$. $N_\alpha$ is the density of the ions and $\alpha_{el}(0)$ and $\alpha_{ion}(0)$ the ionic and electronic polarizibility, respectively, for $\omega = 0$.

Hint: Start with the equation for the dielectric constant in a cubic crystal:

$$
\varepsilon = 1 + \frac{N_\alpha \alpha}{1 - \frac{4}{3} N_\alpha \alpha}
$$

With $\alpha = \alpha_{ion} + \alpha_{el}$.

Problem 3: Phonon resonance of the dielectric function (5 points):

The dielectric function (i.e., the permittivity) in the vicinity of an optical phonon is given by

$$
\varepsilon(\omega) = \varepsilon(\infty) + \frac{\varepsilon(0) - \varepsilon(\infty)}{\omega_0^2 - \omega^2 - i\gamma \omega},
$$

where $\gamma$ parametrizes the damping. Calculate, sketch and explain the real and imaginary part of $\varepsilon(\omega)$ for $\varepsilon(0)=10$, $\varepsilon(\infty)=5$, $\omega_0=10^{14}$ s$^{-1}$ and $\gamma=2.5 \times 10^3$ s$^{-1}$. Find the value of $\omega$ for which $\varepsilon(\omega)=0$! Investigate the case $\gamma=0$!