Problem 1: Reflectivity of metal surfaces, Drude model (6 points):

Calculate the reflectivity loss of a light beam with wavelengths $\lambda = 1000 \mu m, 1.0 \mu m, \text{ and } 0.5 \mu m$. The angle of incidence is $90^\circ$ and the mirror surface is coated with silver. Down to which wavelength is such a mirror useable in an optical spectrometer?

(Collision time $\tau = 4.1 \times 10^{-14} \text{ s}$, lattice constant $a = 4.09 \text{ Å}$)

Problem 2: Metallic sodium (5 points):

For metallic sodium calculate the following properties at $T = 300 \text{ K}$:

a) the time $\tau$ between two scattering processes,

b) the Fermi velocity $v_F$,

c) the mean free path,

d) the average electron drift velocity $v_D$ in a static electric field $E = 100 \text{ V/m}$

($\rho_{Na} (300 \text{ K}) = 4.9 \times 10^8 \text{ Ωm}, T_F = 3.7 \times 10^4 \text{ K}, \text{ one conducting electron per atom}$)

Problem 3: Average energy of a Fermi gas (5 points):

The energy-dependent density of states of a Fermi gas with a volume $V$ is given by

$$Z(E) = \frac{V}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \sqrt{E}$$

Therefore the number of electrons $N$ in this Volume $V$ is given by

$$N = \int_{0}^{E_F} Z(E) dE$$

Use this relations to calculate for $T = 0$ the integral energy $U$ of a particle within the Fermi gas

Problem 4: Electrons at the Fermi edge (4 points):

Determine for the metal copper the fraction of electrons whose energy at room temperature ($T \ll T_F$) is larger than $E_F - 2k_B T$. Here, $T_F(Cu) = 8.1 \times 10^7 \text{ K}$. 