Problem 3: Brillouin zone (3 points):
For a tetragonal crystal (primitive Bravais lattice \( c/a = 1.5 \)) construct the \( k_x - k_z \) plane of the first two Brillouin zones. The Fermi surface of the free electrons is an ellipsoid with the axis 1.5 \( \tilde{A} \) and 1.5 \( \tilde{C} \). In the reduced zone scheme determine the part of the Fermi surface that belongs to the second zone.

Problem 2: Valence band of semiconductors (6 points):
For \( k \) near 0, the valence band of many semiconductors (for example Si, Ge, GaAs) is determined by

\[
E(\vec{k}) = \frac{\hbar^2}{2m_0} \left[ -A \cdot k^2 \pm \sqrt{B^2 \cdot k^4 + C^2 \cdot \left(k^2_x \cdot k^2_y + k^2_y \cdot k^2_z + k^2_z \cdot k^2_x\right)} \right]
\]

The two signs correspond to the light- and the heavy-hole band. Determine the tensor of the reciprocal effective mass \( (1/m^*)_{ij} \) for \( \vec{k} \parallel [110] \) in the limit \( k \rightarrow 0 \). Calculate the tensor for silicon with \( A = 4.29 \text{ Å}, \ B = 0.68 \text{ Å} \) and \( C = 4.87 \text{ Å} \).

Problem 3: Heavy and light holes (5 points):
A non-degenerate semiconductor is described by a parabolic conduction band (effective electron mass: \( m^* \); minimum energy at \( k = 0 \): \( E_C \)) and two parabolic valence bands (effective hole masses: \( m_+ \) and \( m_- \); maximum energy of both bands at \( k = 0 \): \( E_V \) ). Calculate the concentration of heavy and light holes \( (p_+ \) and \( p_- \)) from the neutrality condition \( n = p_+ + p_- \).